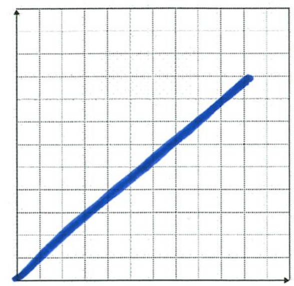
Types of Relations

There are infinite ways that two quantities can be related, however in this course we will be primarily interest in 5 different relations.

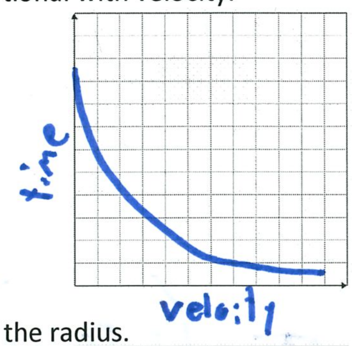
1) A quantity may be **directly proportional** (linear) with another.
   - Example: Distance travelled is proportional to the time travelled if you move at a constant velocity.

$y_1 \sim ax_1 + b$ $\quad$ $d = mt$



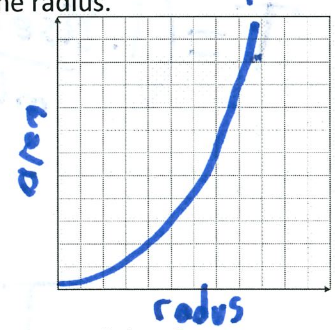
2) A quantity may be **inversely proportional** with another
   - Example: Time to travel a certain distance is inversely proportional with velocity.

$y_1 \sim \frac{a}{x_1} + b$ $\quad$ $t = \frac{m}{v}$



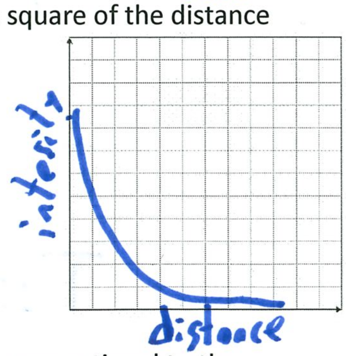
3) A quantity may be **proportional with the square** of another
   - Example: The area of a circle is proportional to the square of the radius.

$y_1 \sim ax_1^2 + b$ $\quad$ $A = mr^2$



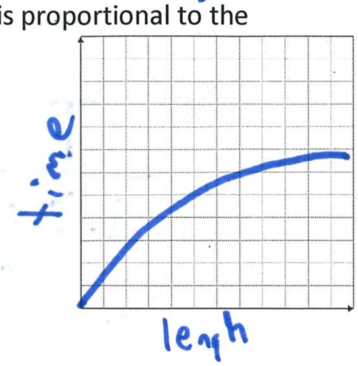
4) A quantity may be **inversely proportional with the square** of another.
   - Example: The intensity of light is inversely proportional to the square of the distance from the source.

$y_1 \sim \frac{a}{x_1^2} + b$ $\quad$ $I = \frac{m}{d^2}$



5) A quantity may be **proportional with the square root** of another.
   - Example: The time it takes a pendulum to go back and forth is proportional to the square root of length of the pendulum.

$y_1 \sim a\sqrt{x_1} + b$ $\quad$ $t = m\sqrt{l}$

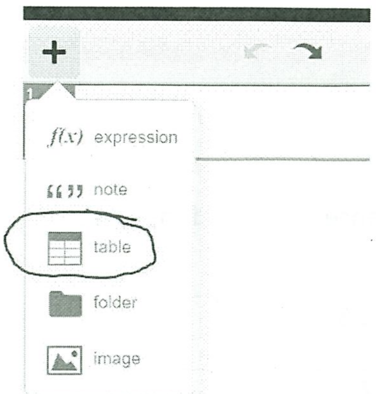


We can use technology to determine the best equation for data.

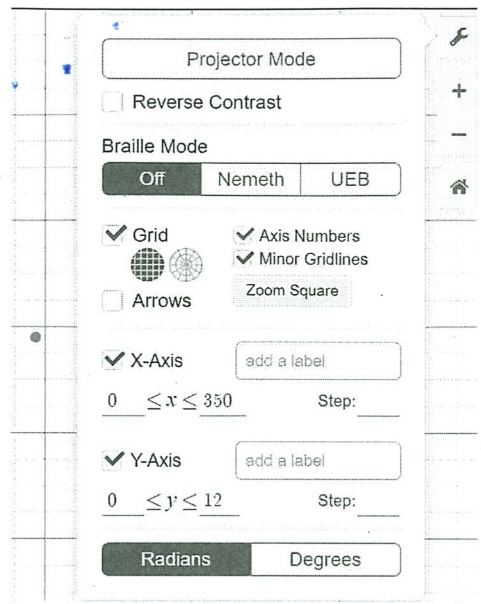
Consider the relationship between height and cargo shown below

Height (m)	11.3	8.5	7.2	5.8	5.4	5.2	5.0
Cargo (tonnes)	52	80	116	197	249	298	324

We can plot this data in Desmos by adding a table, filling in the data, and changing the graphing setting to show the data.

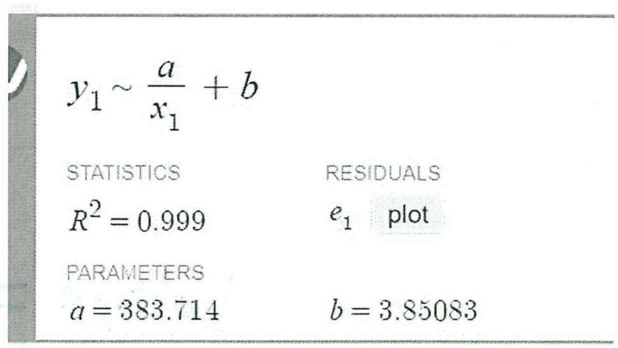


x_1	y_1
52	11.3
80	8.5
116	7.2
197	5.8
249	5.4
298	5.2
324	5.0



Once we have the data, we can determine type of relation by looking at the shape of the graph. In this example it appears height is inversely proportional to cargo.

In the equation field we enter $y_1 \sim \frac{a}{x_1} + b$



We can then determine the equation, in this case it is

$$y_1 = \frac{383.714}{x_1} + 3.85$$

$$H = \frac{380\,t\cdot m}{c} + 3.9\,m$$

In addition we can see the R^2 value, the closer this is to __1__ the better the line describes the data.

Practice: Use Desmos to make a scatter plot, determine the shape of the graph, and then determine the curve of best fit and R^2 value.

- **The Distance Fallen by a Stone vs Time:** From the top of a tall building a stone is dropped. The distance it has fallen is determined by taking photographs every second.

Dependant Variable	Distance(m)	0.0	5.0	19	43	77	120	173	235	307
Independent Variable	Time(s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0

$$d = \left(\frac{4.8m}{s^2}\right) t^2$$

Electric Force vs Distance: Two electric sources are placed different distances apart and the force of attraction is measured.

Dependant Variable	Force (N)	10.0	5.6	3.6	2.5	1.8	1.4	1.1
Independent Variable	Distance(cm)	3.0	4.0	5.0	6.0	7.0	8.0	9.0

$$F = \frac{90 N \cdot cm^2}{d^2}$$

Voltage and Current: The change in voltage as current is increased in measured.

Dependant Variable	Voltage (V)	7.5	12	20	26	35	44	52	56	66
Independent Variable	Current (A)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0

$$Voltage = \left(7.2 \frac{V}{A}\right)(C) - 1.9V$$